

## HARD THERMAL LOOPS IN THE ELECTROWEAK THEORY

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### Abstract

We derive a thermal effective action for soft fields in the broken phase of the electroweak theory in the limit of a strongly interacting Higgs sector. This action is just the proper generalization of the hard thermal loop effective action of a Yang-Mills theory when there is a Higgs mechanism and for a heavy Higgs boson. One can obtain from this action the thermal corrections to the masses of the  $W$ ,  $Z$  and the photon.

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We derive a thermal effective action for soft fields in the broken phase of the electroweak theory in the limit of a strongly interacting Higgs sector. This action is just the proper generalization of the hard thermal loop effective action of a Yang-Mills theory when there is a Higgs mechanism and for a heavy Higgs boson. One can obtain from this action the thermal corrections to the masses of the  $W$ ,  $Z$  and the photon.

In a non-Abelian plasma, such as the one formed in the high temperature  $T$  regime of QCD or in the restored phase of the electroweak theory, the effective theory for the soft modes is given by a sum of the tree level action plus the hard thermal loop one<sup>1</sup>. Soft refers to a physical scale of order  $\sim gT$ , where  $g$  is the gauge coupling constant. The hard thermal loop (HTL) effective action  $S_{HTL}$  is understood as arising after integrating out the modes of order  $\sim T$  of the theory. The effective action  $S_{HTL}$  gives account of the by now well understood phenomena of Debye screening and Landau damping. It has been derived using different approaches in the literature. The first and fundamental one was based on an evaluation of individual Feynman diagrams<sup>1</sup>, and it was essential for all the subsequent developments, as it allowed to recognize the importance of the hierarchy of scales  $T$  and  $gT$  in the plasma. After realizing that all the HTL's were related by simple Ward identities<sup>2</sup>, it was then discovered the underlying gauge symmetry principle of the infinite set of HTL's<sup>3</sup>. The fact that  $S_{HTL}$  is a gauge invariant action was exploited in a subsequent set of papers<sup>4</sup>.

A different language to describe HTL's, that of kinetic theory, was then introduced soon after. Blaizot and Iancu<sup>5</sup> derived a set of kinetic equations from the hierarchy of Schwinger-Dyson equations obtained from the QCD Lagrangian. There is a different approach to yield the same kinetic equations which has as a starting point the Wong equations<sup>6</sup>. The Wong equations are the classical equations of motion of point particles carrying a non-Abelian charge. A transport theory can be constructed from those Wong equations<sup>7</sup>, and in the response theory, again it reproduces the HTL's<sup>8</sup>.

We now understand the physics at soft scales in the symmetric phase of

a hot non-Abelian gauge field theory, and we also have different languages to describe the same thermal physical phenomena. The effective action  $S_{HTL}$  is essentially a mass term for the chromoelectric fields. Chromomagnetic fields are not screened at the same order in the coupling constant, although there is the strong belief that a mass for the magnetic degrees of freedom is generated non perturbatively.

In this talk we will derive an effective action for soft fields in a theory where the non-Abelian gauge fields are already massive at  $T = 0$ . We will then concentrate our attention to the electroweak theory in its broken phase. And more particularly, we will consider the limit where the Higgs field becomes heavy, and then the Higgs sector becomes strongly interacting. If the Higgs boson is very heavy, one can integrate out the Higgs field from the electroweak Lagrangian to finally obtain a low energy effective theory which reads <sup>9</sup>

$$\mathcal{L}_{eff} = -\frac{1}{2}\text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{v^2}{4}\text{Tr}(D_\mu\Sigma D^\mu\Sigma^\dagger) . \quad (1)$$

(see Ref. <sup>10</sup> for notations and conventions).

This low energy effective theory is interesting due to several reasons. The experimental lower bound for the Higgs mass is continuously increasing ( now is above 90 GeV), so the possibility of a heavy Higgs boson is not yet excluded. And even if the Higgs boson does not exist, the above Lagrangian would also describe the low energy theory of no matter what mechanism is responsible for the generation of masses of the gauge bosons. At tree level the model is universal, and thus model independent. The model dependence only arises at the one-loop level. For example, if one wants to include the effects of quantum Higgs boson exchange, additional effective interactions are required, thus <sup>9</sup>

$$\mathcal{L}_4 = L_1\text{Tr}(D_\mu\Sigma D^\mu\Sigma^\dagger)^2 + L_2\text{Tr}(D_\mu\Sigma D^\nu\Sigma^\dagger)^2 , \quad (2)$$

where  $L_{1,2}$  are model dependent constants. The above terms are higher order in an energy expansion.

We will only consider the thermal effects associated to the lowest (universal) effective Lagrangian Eq. (1) after integrating out the hard modes of the theory. We will not consider the effect of fermions for the time being.

In the Lagrangian Eq. (1) there are three types of fields: the weak isospin field  $W_\mu$ , the weak hypercharge field  $B_\mu$  and the would-be Goldstone bosons  $\phi^a$  which are contained in the matrix  $\Sigma$ , and which are unphysical. This is apparent in the unitary gauge  $\Sigma = 1$ , where the Goldstone fields disappear. In this gauge one can easily read the masses of gauge bosons  $W$ , the  $Z$  and the

photon, to reach to the well-known values

$$M_W = \frac{gv}{2}, \quad M_Z = \frac{\sqrt{g^2 + g'^2}v}{2}, \quad M_\gamma = 0. \quad (3)$$

For loop computations it is however much more convenient to work in a different gauge, since in the unitary gauge the gauge field propagators are not well behaved in the infrared region.

To integrate out the hard modes of the theory it is convenient to use the background field method (BFM) <sup>12</sup>. The BFM is a well-known technique to evaluate loop effects, and consists in expanding the tree level Lagrangian around the solution of the classical equations of motion. The one-loop effective action is then obtained after integrating out the quantum fluctuations. The BFM has certain advantages when applied to gauge field theories, as we will see. After the vector gauge fields are split into background (or classical) and quantum pieces, there are two types of symmetries associated to the tree level action. One is associated to the background fields, and the other one is associated to the quantum fields. One can then fix the gauge of the two different symmetries independently, as long as the quantum gauge is fixed in a covariant way with respect to the background gauge symmetry. The one-loop effective action thus obtained is automatically respectful with the background gauge symmetry <sup>12</sup>.

In the spirit of the BFM, one then introduces an additive splitting for the gauge fields, and a multiplicatively one for the matrix  $\Sigma$  <sup>10</sup>. At that point, one can perform a Stueckelberg transformation <sup>13</sup> to get rid of the the background Goldstone fields, which is equivalent to choosing the unitary background gauge, while choosing a different gauge for the quantum fields. In the unitary background gauge one then finds <sup>10</sup>

$$\begin{aligned} S_{eff} + \delta S_{eff,T} = & \quad (4) \\ & = \int d^4x \left\{ -\frac{1}{2} \text{Tr}(\bar{W}_{\mu\nu} \bar{W}^{\mu\nu}) - \frac{1}{4} \bar{B}_{\mu\nu} \bar{B}^{\mu\nu} + \frac{v^2(T)}{4} \text{Tr} \left( g \bar{W}^\mu - g' \bar{B}^\mu \frac{\tau^3}{2} \right)^2 \right\} \\ & - \frac{T^2}{6} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \int d^4x d^4y \text{Tr} \left( \bar{\Gamma}_{\mu\lambda}(x) < x | \frac{Q^\mu Q_\nu}{-(Q \cdot \bar{d})^2} | y > \bar{\Gamma}^{\nu\lambda}(y) \right) \\ & + \frac{g^2 T^2}{3} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \int d^4x d^4y \text{Tr} \left( \bar{W}_{\mu\lambda}(x) < x | \frac{Q^\mu Q_\nu}{-(Q \cdot \bar{D}_W)^2} | y > \bar{W}^{\nu\lambda}(y) \right) \end{aligned}$$

where  $\bar{W}_{\mu\nu}$ ,  $\bar{B}_{\mu\nu}$  are the field strengths of the corresponding background gauge fields, and

$$v(T) = v \left( 1 - \frac{1}{12} \frac{T^2}{v^2} \right), \quad (5)$$

$$\bar{\Gamma}_\mu = \frac{i}{2} \left( g \bar{W}_\mu + g' \bar{B}_\mu \frac{\tau^3}{2} \right) , \quad (6)$$

$$\bar{\Gamma}_{\mu\nu} = \partial_\mu \bar{\Gamma}_\nu - \partial_\nu \bar{\Gamma}_\mu + [\bar{\Gamma}_\mu, \bar{\Gamma}_\nu] , \quad (7)$$

and  $Q = (i, \mathbf{q})$  is a null vector  $Q^2 = 0$ . The angular integral in (4) is done over all directions of the three dimensional unit vector  $\mathbf{q}$ . One can obtain the same action in an arbitrary background gauge just by inverting the Stueckelberg transformation<sup>10</sup>.

The above effective action is the proper generalization of the HTL effective action in the broken phase of the electroweak theory in the limit of a heavy Higgs field. One can obtain the thermal screening masses for all the gauge fields by considering the above action in the static limit. One then finds the following values for the transverse masses

$$M_{W,t}^2(T) = \frac{g^2 v^2(T)}{4} , \quad M_{Z,t}^2(T) = \frac{(g^2 + g'^2) v^2(T)}{4} , \quad M_{\gamma,t}^2(T) = 0 , \quad (8)$$

while for the longitudinal masses one has

$$\begin{aligned} M_{W,l}^2(T) &= \frac{g^2 v^2(T)}{4} + \frac{3g^2 T^2}{4} , \quad M_{\gamma,l}^2(T) = e^2 T^2 \\ M_{Z,l}^2(T) &= \frac{(g^2 + g'^2) v^2(T)}{4} + \frac{(g^2 + g'^2) T^2}{12} (\cos^2 \theta_W - \sin^2 \theta_W)^2 \\ &\quad + \frac{2g^2 T^2}{3} \cos^2 \theta_W , \end{aligned} \quad (9)$$

where  $\theta_W$  is the Weinberg angle. The above results agree with those computed in Ref.<sup>11</sup>. Notice that the transverse and longitudinal modes get different thermal corrections to their masses.

Let us comment the temperature range of validity of the previous results. The above effective action has been derived assuming that  $M_W \ll T \ll \sqrt{12}v$ . The lower limit just tells us that the masses of the gauge bosons are soft. The upper limit is just demanded in order to require that the one-loop effects computed here are small corrections. If this is not the case, then one cannot guarantee that the next to leading order effects are not important. The next to leading order effects would arise by considering two loops from Eq. (1), and the (model dependent) one loop effects generated by Eq. (2).

Let us finally conclude with the observation that it would be very useful to derive kinetic equations which describe these thermal effects in the broken phase of the electroweak theory.

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